Invited Paper

A taste of photonics: band structure, null gaps, non-Bragg gaps, and symmetry properties of one-dimensional superlattices

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Abstract

We have investigated the propagation of plane waves through one-dimensional superlattices composed of alternate layers characterized by two different refractive indexes, which may take on positive as well as negative values. For both indices of refraction positive we have found null-gap points for commensurate values of the optical path lengths of each layer at which the superlattice becomes transparent. We have determined the symmetry properties of the electromagnetic field demonstrating the degeneracy of the solutions at these points. Furthermore, we have been able to characterize non-Bragg gaps that show up in frequency regions in which the average refractive index is null, by obtaining analytically the non-Bragg gap width which depends only on the ratio $\frac{b}{a}$ of the layer widths.

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Wave propagation in periodic media has been for a long time the object of study of many researchers. The notion of a photonic band gap in one dimension has been introduced as early as 1887 by Lord Rayleigh in his investigation on laminated one dimensional structures [1]. Recent advances in nano-fabrication of photonic crystals (PCs) have enabled a number of nanophotonic experiments involving these subwavelength dielectric structures evidencing many of its important features such as the existence of a complete band gap, the construction of ultra-compact light wave circuits and the control of spontaneous emission. The microstructuring techniques of high quality optical materials available nowadays yield to a remarkable flexibility in the fabrication of the photonic crystals, resulting in the tailoring of the electromagnetic dispersion relation and mode structure to suit almost any need, opening new perspectives for both basic and technological research purposes. As an example, negative refractive effects may be exhibited by photonic crystals with enginereed novel dispersion relationships in particular those for which the frequency disperses negatively with the wave vector. This possibility turns the PCs into a fertile ground to explore the exciting consequences provided by a medium exhibiting simultaneously negative permittivity and negative permeability once idealized by Veselago [2]. In his seminal paper, Veselago deduced that the propagation of electromagnetic radiation through such a medium should exhibit backward wave propagation, reversed refraction, reversed Doppler shift as well as reversed Cherenkov radiation. One of those consequences, focusing, has been observed in photonic crystals [3, 4].

The existence of photonic bands in the energy spectrum as well as photonic band gaps have permitted quite a number of analogies between the scattering of particle waves in periodic potentials and the scattering of optical waves propagating in periodic media. In spite of the mathematical analogy, in general the two phenomena are not exactly parallel. Thus, there are optical aspects that have no important counterparts in electronic transport such as for example, the scaling of the lattice parameter. Furthermore, the differences that might exist such as, additional degeneracies or even non-Bragg gaps, as we show in the following, reflect underlying symmetry particularly important to optical phenomena.

We consider incident light, either s polarized or TE (field $\mathbf{E}$ transversal to the incidence plane) or p polarized (field $\mathbf{H}$ transversal to the incidence plane), on a one-dimensional super lattice periodic in the z coordinate according to a repeating stack of N dielectric stacks which alternate thicknesses a and b with $a + b = d$ and indices of refraction n_1 and n_2. We choose x to represent the lateral direction and z the stacking direction so that

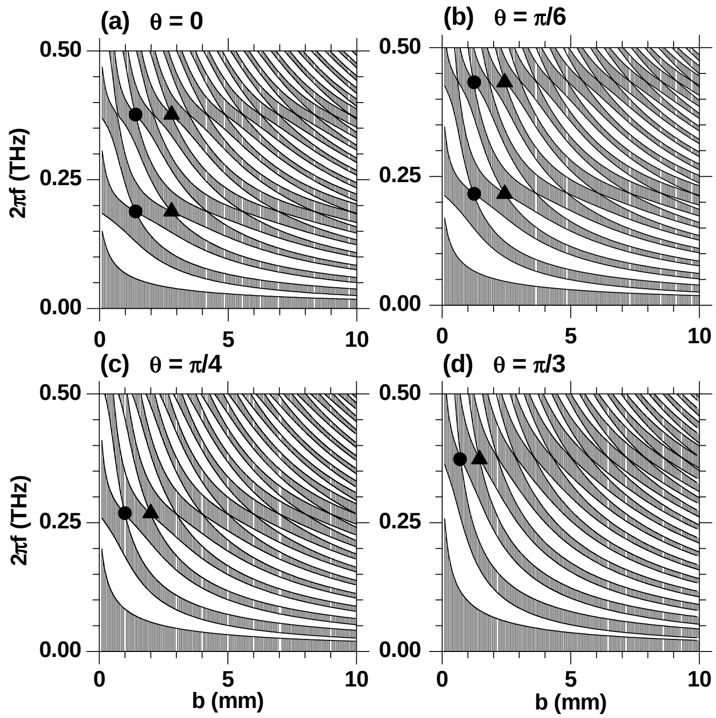

FIG. 1: Photonic bandwidth of a one-dimensional superlattice with alternate layers of air (thickness a and $n_1 = 1$) and GaAs (thickness $b = 5$ mm and $n_b \simeq 3.6$). Results are for $a = 5$ mm and various GaAs layer widths, and illustrate the presence of gaps (white regions) in the band structure for four angles of incidence as indicated in the figure. Notice black symbols representing the occurrence of null band gaps.

$k_{1(2)}^2 = \frac{\omega^2}{c^2} n^2 = k_{1(2)z}^2 + k_x^2$ where $k_x = \frac{n\omega \sin\theta_1}{c}$ is the always real parallel component of the wave vector which is conserved throughout the structure, and θ_1 is the angle between $\mathbf{k}_1$ and the normal to the incidence plane. We are interested in the transmission properties of the superlattice and therefore we suppose that light is incident and detected in medium 1 for which one has real permissivity ε_1 and permeability μ_1 in such a way that $\varepsilon_1\mu_1 > 0$ while ε_2 and μ_2 are real. Let us begin by writing up the transformation matrix that connects points at one layer to those at the neighboring layer by choosing the origin of coordinates located at the center of a first slab of width a, that is [5, 6],

$$\psi(\pm\frac{a+b}{2}) = T(\pm a, \pm b)\psi(0) \tag{1}$$

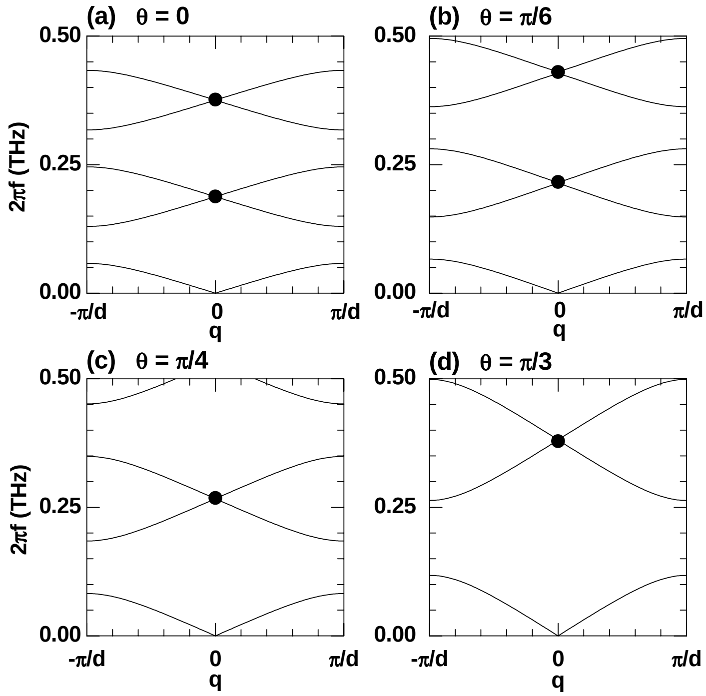

FIG. 2: Dispersion curves illustrating the null gap points (full dots) shown in Fig. 1

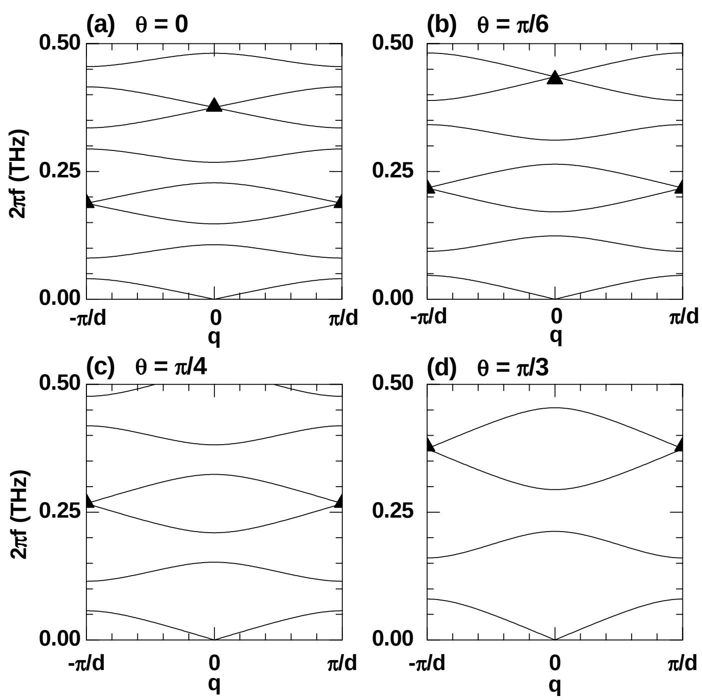

FIG. 3: Dispersion curves illustrating the null gap points (full triangles) shown in Fig. 1

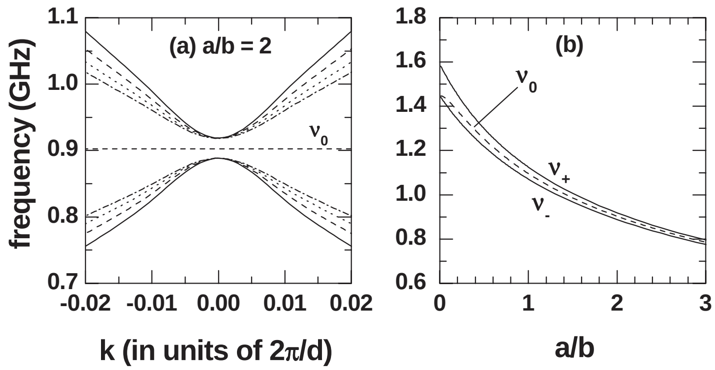

FIG. 4: (a) Dispersion curves exhibiting the invariance of the $< n >= 0$ gap for $\frac{a}{b} = 2$ which opens around $\nu_0 = 0.9$ GHz. Solid, dashed, dotted, and dash-dotted lines correspond to $a = 12$ mm, 14 mm, 16 mm and 18 mm, respectively; (b) $\frac{a}{b}$ - dependence of ν_0 and ν_-/ν_+ frequencies corresponding to the top/bottom of the first/second photonic bands in (a). Results are from Cavalcanti *et al* [6].

where

$$T(\pm a, \pm b) = M_2\left(\pm\frac{b}{2}\right) M_1\left(\pm\frac{a}{2}\right) = \begin{pmatrix} p(\omega) & \pm q(\omega) \\ \pm r(\omega) & s(\omega) \end{pmatrix}, \tag{2}$$

and

$$P = \cos\frac{bk_{2z}}{2}\,\cos\frac{ak_{1z}}{2} - \frac{Z_2 n_2}{Z_1 n_1}\,\frac{k_{1z}}{k_2}\,\sin\frac{bk_{2z}}{2}\,\sin\frac{ak_{1z}}{2}\,, \tag{3}$$

$$Q = \frac{Z_1 n_1}{k_{1z}}\,\cos\frac{bk_{2z}}{2}\,\sin\frac{ak_{1z}}{2} + \frac{Z_2 n_2}{k_{2z}}\,\sin\frac{bk_{2z}}{2}\,\cos\frac{ak_{1z}}{2}\,, \tag{4}$$

$$R = -\frac{k_{2z}}{Z_2 n_2}\,\sin\frac{bk_{2z}}{2}\,\cos\frac{ak_{1z}}{2} - \frac{k_{1z}}{Z_1 n_1}\,\cos\frac{bk_{2z}}{2}\,\sin\frac{ak_{1z}}{2}\,, \tag{5}$$

$$S = \cos\frac{bk_{2z}}{2}\,\cos\frac{ak_{1z}}{2} - \frac{Z_1 n_1}{Z_2 n_2}\,\frac{k_{2z}}{k_{1z}}\,\sin\frac{bk_{2z}}{2}\,\sin\frac{ak_{1z}}{2}\,. \tag{6}$$

The dispersion relation of such a structure may be computed after using the transfer matrix and imposing the Bloch condition $E(z + d) = e^{iqz}E(z)$, with q as the Bloch vector defined within $-\frac{\pi}{d} \leq q \leq \frac{\pi}{d}$, which is obtained from the eigenvalues of the transfer matrix corresponding to a single bilayer. The following dispersion relations for TE and TH polarizations, respectively, reads [5]:

$$\cos qd \;=\; \cos ak_{1z}\cos bk_{2z} - \frac{1}{2}\left[\frac{\mu_1 k_{2z}}{\mu_2 k_{1z}} + \frac{\mu_2 k_{1z}}{\mu_1 k_2}\right]\sin ak_{1z}\sin bk_{2z}\,, \qquad (TE) \tag{7}$$

$$\cos qd \;=\; \cos ak_{1z}\cos bk_{2z} - \frac{1}{2}\left[\frac{\varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z}} + \frac{\varepsilon_2 k_{1z}}{\varepsilon_1 k_{2z}}\right]\sin ak_{1z}\sin bk_{2z}\,. \qquad (TH) \tag{8}$$

Let us now suppose a superlattice composed by media with frequency independent positive refraction indexes n_1 and n_2. To investigate the effect of the variation of relative air and $GaAs$ widths, we have plotted in Fig.1 the bandwidths, in the case of an air layer with $a = 5\,mm$, as functions of the $GaAs$ layer thickness, for four angles of.incidence illustrating the presence of the band gaps as well as of null gap points denoted by the black circles and triangles. For normal incidence, one may show [6] that the occurrence of null gaps is at $\frac{b}{a} = \frac{n_1}{n_2}\frac{N_2}{N_1}$, with $N_1, N_2 = 1, 2, 3, ...$, and at $\omega = \frac{N_1 \pi c}{n_1 a} = \frac{N_2 \pi c}{n_2 b}$. Note that the touching of the bands occurs when $R(\omega) = Q(\omega) = 0$ at the BZ-center, and when $P(\omega) = S(\omega) = 0$ at the BZ-edge, implying the following zero photonic band-gap conditions, $\frac{a\omega n_1}{c} = N_1\pi$ and $\frac{b\omega n_2}{c} = N_2\pi$, where N_1 and N_2 are integers (alternatively, one may write $a = N_1\lambda_1/2$ and $b = N_2\lambda_2/2$). In terms of optical paths one has for the particular case of normal incidence:

$$\frac{n_2 b}{N_1} = \frac{n_1 a}{N_2}, \tag{9}$$

so that the optical paths must be commensurable at the null gap points. At the BZ-center N_1 and N_2 are both even or both odd, while at the BZ-edge N_1 and N_2 are of opposite parities. In Figs. 2 and 3 we have plotted the dispersion curve in order to illustrate the null gap points occurring at the center of the BZ and at the edge of the BZ. It is clear from Fig. 2(a) that the dispersion curve may be dramatically modified, exhibiting a band touching at the center of the Brillouin zone [6], with a finite derivative $\frac{d\omega}{dq} = \pm \frac{d}{2\sqrt{\gamma}}$, where $\gamma = \frac{1}{4}[(1 + \frac{Z_2}{Z_1})(1 + \frac{Z_1}{Z_2})\alpha^2 + (1 - \frac{Z_2}{Z_1})(1 - \frac{Z_1}{Z_2})\beta^2]$, $\alpha = \frac{1}{2}(\frac{an_1}{c} + \frac{bn_2}{c})$ and $\beta = \frac{1}{2}(\frac{an_1}{c} - \frac{bn_2}{c})$. The electric field configurations for normal incidence may be obtained and reads [6],

$$\begin{aligned} E(z) &= E(0)\left[\cos k_1 z + i\frac{\sin qd}{2QS}\frac{\mu_1}{k_1}\sin k_1 z\right], \ |z| < \frac{a}{2} \\ E(z) &= E(0)\left[\left(P + i\frac{\sin qd}{2S}\right)\cos k_2\left(z - \frac{d}{2}\right) + \frac{\mu_2}{k_2}\left(R + i\frac{\sin qd}{2Q}\right)\sin k_2\left(z - \frac{d}{2}\right)\right], \ \frac{a}{2} < \ z < \frac{a}{2} + b \end{aligned} \tag{10}$$

with $E(0)$ as the electric field at the center of the first slab. These equations are useful when $Q(\omega)$ and $S(\omega)$ are different from zero, while the following are useful

$$\begin{aligned} E(z) &= E(0)\left[\frac{\mu_1}{k_1}\sin k_1 z + i\frac{\sin qd}{2PR}\cos k_1 z\right], \ |z| < \frac{a}{2} \\ E(z) &= E(0)\left[\left(Q + i\frac{\sin qd}{2R}\right)\cos k_2\left(z - \frac{d}{2}\right) + \frac{\mu_2}{k_2}\left(R + i\frac{\sin qd}{2P}\right)\sin k_2\left(z - \frac{d}{2}\right)\right], \ \frac{a}{2} < \ z < \frac{a}{2} + b \end{aligned} \tag{11}$$

when $P(\omega)$ and $R(\omega)$ are non-zero. The above equations provide the electromagnetic modes in a single elementary cell of the direct photonic crystal. By using the Bloch condition one may determine these modes in other elementary cells. The symmetry properties of the electromagnetic modes are: (i) at the BZ-center ($k = 0$), they are even functions of z when $R(\omega) = 0$ and $Q(\omega) \neq 0$, whereas they are odd when $R(\omega) \neq 0$ and $Q(\omega) = 0$; ii) at the BZ-edge ($k = \pm\pi/d$), these modes are even if $S(\omega) \neq 0$ and $P(\omega) = 0$, whereas they are odd when $S(\omega) = 0$ and $P(\omega) \neq 0$; iii) the touching of the bands occurs for $R(\omega) = Q(\omega) = 0$ at the BZ-center, and for $P(\omega) = S(\omega) = 0$ at the BZ-edge, so that the solutions are degenerate and the general solution has no definite parity. To investigate further the physical consequences of this degeneracy, we proceed to the computation of the coefficient of transmission t (reflection r) which is defined as the ratio of the transmitted (reflected) amplitude at the input to the incident field amplitude considering that incidence is from the left. Introducing the $\frac{r}{t}$ ratio of reflected amplitude to the transmitted amplitude for $N = 1$, for normal incidence [5],

$$\frac{r}{t} = \frac{i}{2}\left(\frac{k_1}{\mu_1} - \frac{k_2}{\mu_2}\right)\sin(k_2 b)e^{ik_1 a}\,, \tag{12}$$

the transmission for a superlattice of N layers may be written as [5]:

$$T = \frac{1}{1 + \left|\frac{r}{t}\right|^2 \left|\frac{\sin(Nqd)}{\sin(qd)}\right|^2}\,. \tag{13}$$

By using (8), one finds that $\frac{r}{t} = 0$, hence $T = 1$ irrespective of N. At the null-gap points, light behaves as if travelling by a homogenous medium as it can be verified by plugging the null-gap points requirement into the transference matrix. The latter becomes the identity matrix at these particular points and the dispersion curve becomes a straight line with a finite derivative as it is illustrated in Figs. 2(a) and 3(a). In contrast with the Bragg mirrors, the interference between the light arriving from each interface is destructive, there is no light reflected and the layered structure becomes completely transparent.

Let us now deal with a superlattice whose unit cell is composed by a medium with non-dispersive positive n_1 and another one with a dispersive negative $n_2(\omega)$. To include a negative index of refraction, one must take into account that media with negative ε and μ are necessarily dispersive and dissipative and we choose a medium where the effective dielectric permittivity and magnetic permeability are given by [7],

$$\varepsilon(\omega) = \varepsilon_0 - \frac{\alpha}{\omega^2} \tag{14}$$

$$\mu(\omega) = \mu_0 - \frac{\beta}{\omega^2} \tag{15}$$

where $\alpha, \beta, \varepsilon_0$ and μ_0 are positive constants. Let us now define an average index such as,

$$< n(\omega_0) >= \frac{1}{d}\int_0^d n(z,\omega_0)dz = 0\,, \tag{16}$$

so that for the particular case of zero average one finds that

$$\frac{an_1 - b\,|n_2(\omega_0)|}{d} = 0 \tag{17}$$

indicating that light propagating with a particular frequency ω_0 feels in average, a null index of refraction. A gap around this particular frequency is known to be formed, which in contrast with the usual Bragg gap, is insensitive to the size of the unit cell [8], as it is depicted in Fig. 4(a) where the dispersion curves around the gap is plotted. Upon using the macroscopic response functions ε and μ one must have in mind that the wavelength of the radiation must be much larger than the size of the unit cell, i.e., $\lambda_i = \frac{2\pi c}{|n_i|\omega} >> a, b$. Provided that a and b are in the range of $\approx$ 1-20 mm, one may therefore show [6] that, at the $k = 0$ BZ - center, the ν_- frequency, corresponding to the even solution associated with the top of the lower band around the $< n >= 0$ gap illustrated in Fig. 4 (a), is given by the solution of $R(\omega) = 0$, i.e.,

$$\frac{\varepsilon_1}{\varepsilon_2(\omega)} = -\frac{b}{a}\,. \tag{18}$$

Using Eqs. (14) and (15), and the fact that here medium 1 is filled with air so that $\varepsilon_1 = 1$ and $\mu_1 = 1$, one finds

$$\nu_- = \frac{1}{2\pi}\sqrt{\frac{\alpha}{(\varepsilon_2 + \frac{a}{b})}}\,. \tag{19}$$

Analogously, one may show that the ν_+ frequency, corresponding to the odd solution associated with the bottom of the higher band around the $< n >= 0$ gap in Fig. 4(a), may be derived from $Q(\omega) = 0$ which yields to

$$\frac{\mu_1}{\mu_2(\omega)} = -\frac{b}{a}\,. \tag{20}$$

Substituting (14) and (15), and the air values, one finds

$$\nu_+ = \frac{1}{2\pi}\sqrt{\frac{\beta}{(\mu_2 + \frac{a}{b})}}\,. \tag{21}$$

Notice that, as it happenned with the ν_0 frequency associated with the $< n >= 0$ gap, both ν_- and ν_+ are only dependent on the $\frac{a}{b}$ ratio. In that respect, Fig. 4(b) displays the gap profile as a function of the relative layer width, i.e., the $\frac{a}{b}$- dependence of ν_0, and of the ν_-/ν_+ frequencies corresponding to the top/bottom of the first/second photonic bands displayed in Fig. 4(a) [here we note that a full calculation with a and b are in the range of $\approx$ 1-20 mm gives essentially the same results as in Fig. 4 (b)].

To conclude, we have analyzed the photonic band structure of one-dimensional periodic arrays composed of two layers of refractive indexes n_1 and n_2, which may take on positive as well as negative values. Within a transfer-matrix formalism, we have obtained the photonic band structure and electromagnetic profiles. We have found new features and confirmed predicted ones that are inherent to optical phenomena involving the propagation of light, i.e., null-gap points that appear whenever the optical path lengths within each layer are commensurate. In that respect, we have found a condition closely related to the well-known occurrence of the so-called Bragg mirrors except that here, instead of a constructive interference one finds destructive interference, there is no light reflected and the medium becomes transparent irrespective of the number of layers. The transfer matrix at those points becomes the identity matrix so that at these points, light behaves as if travelling through an homogeneous medium, a fact that is translated by a linear dispersion relation and hence by a finite density of states. Finally, we have characterized non-Bragg gaps, i.e., gaps that do not exhibit the usual Bragg sensitivity to cell sizes and that show up in frequency regions in which the average refractive index is null. We have found analytically the gap width and demonstrated that it depends only on the $\frac{a}{b}$ ratio in contrast with usual Bragg gaps. As these are induced by an average effect, the notion of non-Bragg gaps could be extended to a higher class of inhomogeneous materials, as long as the inhomogeneities occur on length-scales much smaller than the wavelength of the radiation, but large compared with atomic or molecular length-scales.

Acknowledgments

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